National Numeracy
for everyone, for life

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Lywodraeth Cymru
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National Support Programme

Llythrennedd a Rhifedd
Literacy and Numeracy

Leadership guide

Developing numerical reasoning:
more knowing, less remembering

Rhaglen Gymorth Genedlaethol
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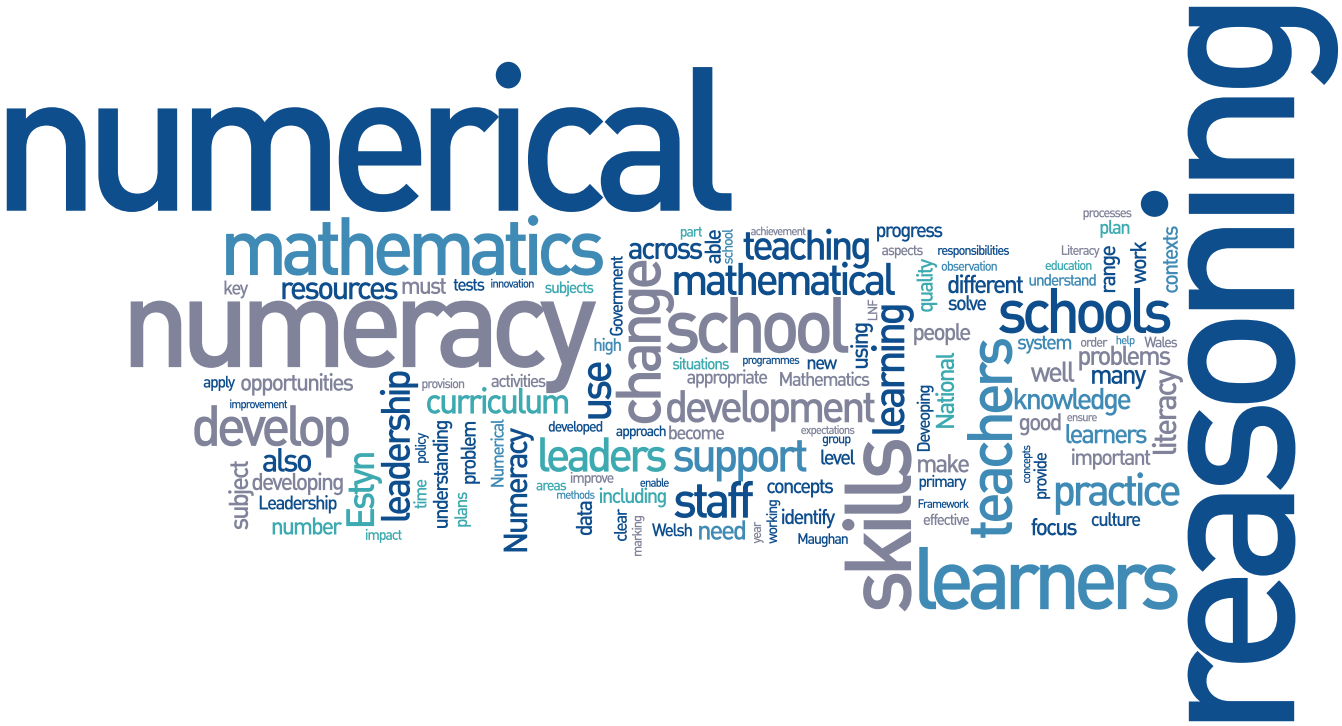

These materials have been carefully developed for use in schools. We are however always grateful for suggestions about how they might be improved or the language modified to make them as accessible as possible for teachers. If you have any suggestions or comments, please send them to dmcgowan@cfbt.com

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Introduction

This leadership guide is written for those who want to understand the concepts of numeracy and numerical reasoning, and their place in schools. The leadership guide will respond to these central issues and will go on to describe steps that schools should take to introduce, develop and embed effective learning and teaching for numerical reasoning in the Foundation Phase, Key Stage 2 and Key Stage 3.

Mathematics, numeracy, numerical reasoning: how do these relate to one another?

Mathematics is defined as:

- *a group of related sciences, including algebra, geometry, and calculus, concerned with the study of number, quantity, shape, and space and their interrelationships by using a specialised notation;*
- *the systematic treatment of magnitude, relationships between figures and forms, and relations between quantities expressed symbolically*.

(Dictionary.com)

The relationship between mathematics and numeracy is set out in the information document no: 120/2013: *National Literacy and Numeracy Framework. To support schools in introducing the National Literacy and Numeracy Framework:*

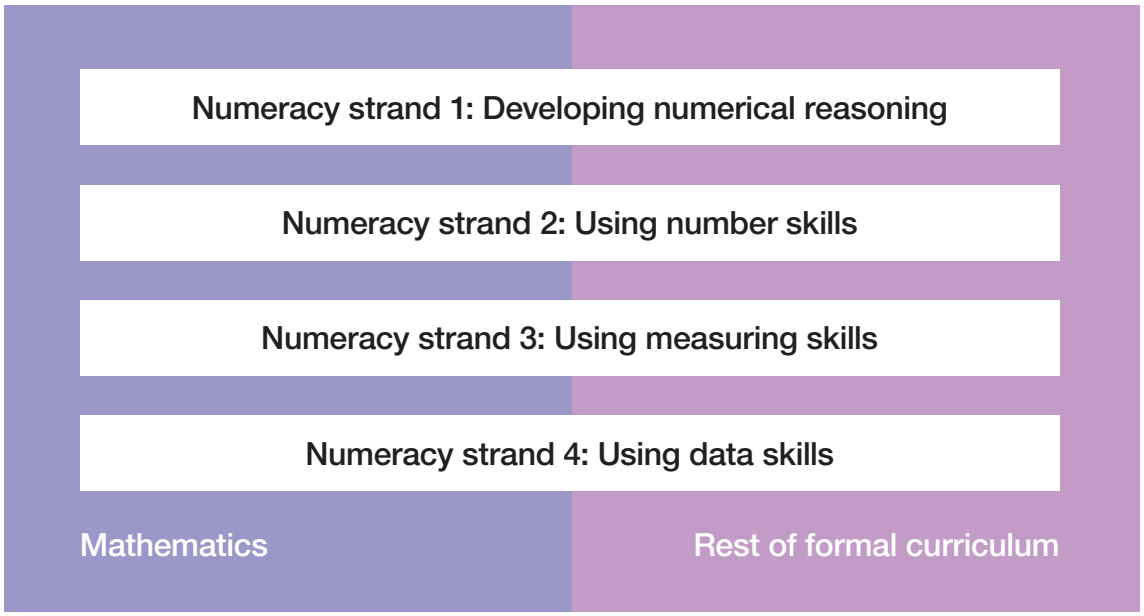
"Numeracy refers to the application of mathematical understanding in daily activities at school, at home, at work, and in the community. There is more to numeracy than teaching the rules and procedures of mathematics. However, it is imperative that the fundamental mathematical techniques are taught to a standard that allows learners to be numerate. Numeracy describes the set of skills needed to tackle real-world problems in a variety of situations by applying numerical reasoning in order to plan how to solve the problem, and then carrying out the mathematical procedures to find the solution.

Numeracy is different to the mathematics subject in that it is the application of the skills learned in mathematics in a cross-curricular, real-world way, and not purely about the skills themselves. The numeracy component of the LNF reflects this, especially in Key Stage 3, where it is not expected that the procedure skills need to become much more advanced, but the context and problem will become more difficult and so the reasoning skills (e.g. knowing how to solve the problem) are expected to develop."

(Welsh Government, 2013a: 20)

Figure 1 illustrates the way that mathematical skills, knowledge and understanding should be used in mathematics **and also** across the rest of the curriculum.

Figure 1 Numerical reasoning within numeracy across the whole curriculum



In Estyn's view, *"Numeracy is not the same as mathematics. Numeracy is a proficiency with number that is acquired through being taught mathematics well. Although learners usually learn their numeracy skills during mathematics lessons, to be fully numerate they must be able to apply these skills in other subject areas and real-life contexts."* (Estyn, 2011b:6)

The Welsh Government describes the relationship between numeracy and numerical reasoning when it states that numeracy is: *"Identifying and applying numerical reasoning skills in order to solve a problem, and carrying out the numerical procedures which enable people to work out and show their solutions."* (Welsh Government, 2012a:2)

It was clear from the pre-tests in 2013 that learners, and many teachers, were unfamiliar with the need to reason numerically. Therefore, strand 1 of the Numeracy component of the Literacy and Numeracy Framework (LNF), 'Developing numerical reasoning', must be an explicit focus for school leaders from 2014.

What is numerical reasoning?

A Background Paper – *Numerical reasoning in the Literacy and Numeracy Framework in Wales 2013: What is numerical reasoning and how can teachers support children to develop it?* (National Numeracy, 2014) – provides a thorough analysis of the issues. It develops, through references to a broad evidence base, definitions of numerical reasoning and of 'developing numerical reasoning'.

See Appendix 1 for the Executive summary of the Background Paper.

Those definitions are set out here:

*"**Numerical reasoning** is the process of using 'number sense' (i.e. knowing when to use a particular operation; when to use mathematics relationships; ability to monitor one's performance when computing, for example, judging reasonableness of answer with respect to an applied problem or by what one knows about numbers) which facilitates the formation of conclusions, judgments, or inferences from facts or premises in order to tackle real-world mathematical problems in a variety of situations. For numerical reasoning to be possible, it is necessary that 'powers' are developed and brought to the surface; also that learners become accomplished at 'knowing-to', so that they are able to respond flexibly and fluently to problems.*

***Developing numerical reasoning** is teaching and learning that guides and supports the development of skills and conceptual mathematics knowledge such that learners become increasingly able to 'know-to' and 'know-how-to' use their interconnected knowledge and their well developed problem-solving skills in order to tackle and solve real-world problems."* (National Numeracy, 2014:2)

Since numeracy clearly requires numerical reasoning it is right and proper that the development of numerical reasoning should be a key focus for teachers, within mathematics and across the whole curriculum. It is numerical reasoning that enables learners' 'sense-making' by using mathematical thinking and this is what gives learners access to new understandings, in mathematics and all other subjects too.

Why does it matter?

Statistics show that learners who are not numerate – that is, they do not have an 'at-homeness' with number (Cockcroft 1982: §39); do not possess core arithmetical skills; are not able to apply mathematics in real-world contexts – are disadvantaged in many ways as they grow up and into adulthood.

National Numeracy (http://www.nationalnumeracy.org.uk/what-the-research-says/index.html), launched in England in 2012, sets out a long and bleak list of the ways in which innumeracy is likely to influence lives and livelihoods, including:

"For women, poor numeracy is an independent predictor of:

- *poor physical health*
- *depression*
- *a perceived lack of control over their lives*
- *being out of the labour market (regardless of how many children they had) or, if in work, of being in an unskilled or semi-skilled job*
- *living in a household where no-one works.*

For men, even when their literacy is good, poor numeracy is associated with:

- *less likelihood of participation in a company pension scheme*
- *higher risk of depression*
- *higher probability of exclusion from school and of arrest by the police"*

It is clear that the development of numerate learners should be an aspiration for education so that school leavers are equipped to be successful adults. However, international comparison, through the Programme for International Student Assessment (PISA), was dispiriting in 2009 when PISA data showed very clearly that 15-year-olds in Wales did not achieve well in mathematics compared to their peers internationally. At the time Leighton Andrews referred to it as "*a wake-up call*". (Welsh Government, *Improving schools*, 2012b)

The PISA mathematics assessment is "*concerned with the reproduction of mathematical knowledge; and in addition, in solving the PISA assessment tasks, students are typically required to extrapolate from what they have learned in school and to apply mathematical knowledge to authentic problems situated in a variety of contexts.*" (OECD, 2009:84)

The tests, taken every three years, are intended to challenge students' knowledge and capability to apply it in a range of problem situations. OECD explains: "*A crucial capacity implied by [this] notion of mathematics is the ability to pose, formulate, solve and interpret problems using mathematics within a variety of situations and contexts. The contexts range from being purely mathematical to having no mathematical structure present or apparent at the outset – the problem poser or solver must*

successfully introduce the mathematical structure. It is also important to emphasise that the definition is not just concerned with knowing mathematics at some minimal level; it is also about doing and using mathematics in situations that range from the everyday to the unusual, from the simple to the complex." (OECD, 2009:84)

The questions in PISA tests are not simple. That is because the aim of these tests is the assessment of problem-solving – involving multi-step problems and the application of a range of mathematical skills and understandings. The link with numerical reasoning is clear. Moreover, it is also clear that in many countries, 15-year-olds are more successful than those in Wales. It is appropriate that the curriculum and teaching in Wales must develop students' capacity to solve problems mathematically, using numerical reasoning and processes.

Wales' PISA performance has declined from 2006:

Year tests taken	Ranking in mathematics
2006	33rd of 59 countries
2009	40th of 67 countries
2012	43rd of 65 countries

In 2012, for mathematics, 15-year-olds in Wales scored 468 points on average, compared with 498 in Scotland, 495 in England and 487 in Northern Ireland (mean = 500).

Part of the *Improving schools* agenda, introduced to combat this decline, has been the introduction of the LNF, now statutory.

In 2013 the first statutory tests for numeracy were introduced, focusing solely on procedural knowledge. The first statutory tests for numerical reasoning will be introduced for all learners in Years 2–9 in 2014. The reasoning tests were trialled in May 2013 amongst a sample of schools, and the Welsh Government published a *Summary report on the numeracy reasoning pre-tests taken in May 2013* (Welsh Government, 2013b). The report describes how the tests were trialled and analysed, and explains that "*As these tests are so new in style, we anticipated that the results were likely to be disappointing*." It goes on to report: "*Mean marks varied but were all disappointing, e.g. 25% (year 3 test A1), 26% (year 4 test A2), 29% (year 5 test B1), 28% (year 7 test B2) and 19% (year 9 test B1)*", and goes on to outline the factors that they believe contributed to the results.

How can schools and teachers support learners to develop numerical reasoning?

How does numerical reasoning develop?

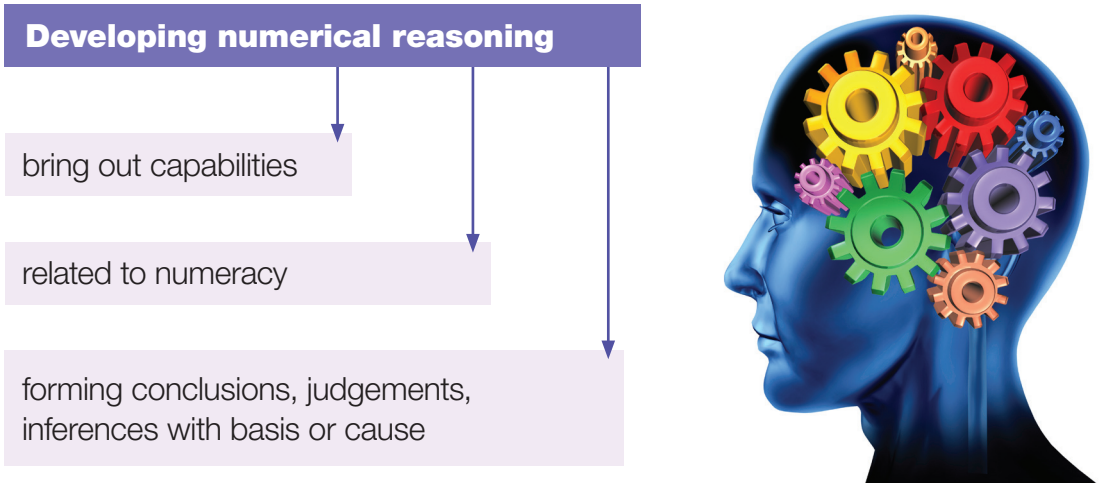
Current reforms within the education system in Wales mean that schools need to show that they are effectively implementing the LNF which sets out expectations for all year groups up to Year 9, in four strands for numeracy.

Strand 1: 'Developing numerical reasoning' is set out in age bands, rather than single year groups and does not, therefore, provide detailed support for teachers. A detailed Progression Chart (available online) has been developed to break down the expectations by year group and by element.

Our definition of 'Developing numerical reasoning', as stated previously, is: "*teaching and learning that guides and supports the development of skills and conceptual mathematics knowledge such that learners become increasingly able to 'know-to' and 'know-how-to' use their interconnected knowledge and their well-developed problem-solving skills in order to tackle and solve real-world problems*." (National Numeracy, 2014:2)

For this development to take place, problem-solving skills and personal dispositions and capacities must be 'brought out' and encouraged to become part of every learner's toolkit for numeracy.

Figure 2 What does 'Developing numerical reasoning' actually mean?



National Numeracy has published its view of the 'Essentials of Numeracy' and an explanation of being numerate. This illustrates how problem-solving, reasoning and decision-making are all vital elements for numeracy. Underpinning all of those is a set of 'attitudes of mind' – for example perseverance, self-confidence, willingness to try, and being comfortable with numbers – which are also necessary if an individual is to become numerate.

Within the Progression Chart, details for developing learners' capacity to explain, interpret, construct and argue are described. For example, Element 1 is 'Identify processes and connections' and it includes the bullet point 'Identify steps to complete the task or reach a solution'. The Progression Chart recommends that children in Year 2 should be able to:

- choose to 'think aloud' as a strategy for making sense of a problem;
- represent the task or problem using pictures and/or concrete objects;
- with support, identify 'givens, goals and gaps' – What do they know? What do they need to know? How might they obtain that information?

Developing capacities and dispositions

Over the last 15 years, there has been increasing interest and argument about the relevance of our systems of education to a 21st-century world. Many educators and policy influencers believe that a modern curriculum should aim to prepare young people for the pressures of a very different real-world context to the one that a subject-focused, knowledge-based curriculum used to be suited to. For example, Professor Guy Claxton, in his keynote speech to the BERA Conference 2006 entitled 'Expanding the Capacity to Learn: A new end for education?' stated that "… education is fundamentally a preparation for the future, not a veneration of the past. Trying to perfect an education system that is fundamentally designed to give young people things they no longer need is not a responsible pursuit." (Claxton, 2006:3). He described four key learning dispositions that we should aim to develop in young people:

- resilience
- resourcefulness
- reflectiveness
- reciprocity.

He went on to state: "*Another issue to be faced is that learning capacity is as much a matter of character as it is of skill. Being able to stay calm, focused and engaged when you don't know what to do is not merely a matter of technical training. It requires a self-concept that has not been infected by the pernicious idea that 'Being confused and making mistakes means you are stupid'. And gaining and losing such attitudes and beliefs takes time and consistency. Of course learning capacity is partly a matter of skill. But we also need a richer vocabulary that includes words like attitudes, dispositions, qualities, values, emotional tolerances, habits of mind.*" (Claxton, 2006:4)

Cuoco, Goldenberg and Mark considered what they call 'habits of mind' that students should develop if they are to be prepared for the world:

"*If we really want to empower our students for life after school, we need to prepare them to be able to use, understand, control, modify, and make decisions about a class of technology that does not yet exist. That means we have to help them develop genuinely mathematical ways of thinking.*" (Cuoco et al., 1996:401)

They believe that students should be: pattern-sniffers, experimenters, describers, tinkerers, inventors, visualisers and conjecturers.

Both the LNF in Wales, and the Progression Chart aim to set out clearly the steps that make up the journey that learners will need to take in order to develop these behaviours, dispositions and habits of mind that will enable them to be numerate in their own real world as children and as adults.

See course handout 'Extracts from the Progression Chart'.

Find an example of a recommendation on the Progression Chart that you are pleased to see.

Find an example of a recommendation that you are surprised to see. Why?

Why is conceptual understanding important?

Mathematics concepts are developed through experience with the world and through forming, reinforcing and modifying connections between aspects of those concepts. With experience, concepts become rich networks of associated knowledge and experience which become 'understandings'. Through these 'understandings' we develop skills for using and applying those concepts and for making decisions about whether and how to re-use aspects – 'pieces' of our

knowledge. It is therefore crucial that teachers are able to understand how to help learners build strong, rich concepts in mathematics. Learners must then also be guided to situations where they can "*flex their learning muscles*" (Claxton, 2002) in order to practise and further develop mathematical knowledge and skills in a wide range of situations – those that are overtly mathematical and those that do not appear to be mathematical at all. Indeed, often in real-world situations, the mathematics inherent in a problem are not at all obvious. For example, one of the Year 4 activities within the Teacher Support Materials that have been sent to schools is called 'Buying a Scooter' – where learners need to use a calendar to work out when Jane will be able to buy a scooter.

(Learning Wales: Welsh Government Teacher Support Materials)

This is not something that children will necessarily perceive as a mathematical problem and yet they need to be able to apply core arithmetical skills, reason methodically, apply their solutions to the real-world context and communicate their thinking. So, the solution to a fairly simple problem can only be achieved by reasoning numerically and using number skills through a number of steps.

This is a new kind of pedagogy for many teachers and a challenging one – not surprising when we consider that most primary teachers are not mathematics specialists and that their own mathematics knowledge and experiences might be lacking robust, connected concepts. We must, therefore, find ways to develop teachers' knowledge and understanding of mathematics concepts if they are to equip their learners to become numerate and to be able to reason numerically. School leaders will need to work with subject leaders and perhaps with other schools to develop staff – to equip them with the subject expertise (mathematical understanding and pedagogical knowledge) that they must possess in order to support high quality learning.

Developing teachers' conceptual knowledge is equally as important as developing that of learners. Is this something that is reflected in your recruitment and CPD processes?

One source of support for teachers to develop their mathematics-specific expertise is the 'Self-Evaluation Tools' developed by the National Centre for Excellence in the Teaching of Mathematics (NCETM) and provided on their website (www.ncetm.org.uk/self-evaluation/).

Policy and practice

It is now the responsibility of school and system leaders, as well as schools working together, to implement the LNF (and to assess against it from September 2014). The strategic direction from leadership teams in schools must be clear, to drive a coherent and cohesive shift in mathematics pedagogy and high quality teaching for numeracy. The quality of such strategic direction in Welsh schools will be monitored by Estyn, who require inspectors to consider the nature and quality of strategic direction in schools.

"*Inspectors should consider the extent to which leaders have clear aims, strategic objectives, plans and policies that are focused on meeting learners' needs. They should ask whether these plans are appropriately focused and whether they are being implemented and monitored in a timely way.*" (Estyn, 2013b:32)

Estyn provides high quality, relevant guidance and support for school leaders about self-evaluation and high quality teaching for numeracy. Samples of Estyn guidance for inspectors and for schools can be found in Appendices 2–5.

Estyn explains that: "*Effective improvement plans:*

- *are based on the outcomes of self-evaluation;*
- *contribute to self-evaluation processes and to further action planning;*
- *are contained in working documents that help to achieve the required changes;*
- *have clearly identified priorities, targets and timescales, and identify steps which will need to be taken to address issues;*
- *identify persons responsible for actions, timescales and the resources and support that will be needed;*
- *specify clear outcomes related to improvement in standards and quality, against which progress can be measured; and*

- *give details of how, when and by whom progress will be monitored and evaluated.*" (Estyn, 2011a:6)

In 2013, Estyn revised the *Guidance for inspecting Literacy and Numeracy* (Estyn 2013a), which sets out a process for inspection in school. It starts with:

"*From your analysis of the performance data and evidence from the Self Evaluation Report (SER), decide whether literacy or numeracy, or both, are likely to require extra attention due to significant issues that have arisen at the pre-inspection stage.*

You should give extra attention to the inspection of numeracy when:

- *data from the national numeracy tests indicate slow/little progress on numeracy;*
- *book scrutiny does not show enough evidence of learners applying their numeracy skills across the curriculum;*
- *SER and School Development Plan (SDP) indicate that the provision for numeracy is a low school priority; and*
- *key indicators for mathematics are low when compared with family, FSM benchmarks and prior attainment.*" (Estyn, 2013a:5)

It is essential that schools can provide evidence of learners' achievement in numeracy – which includes specific consideration of the development of numerical reasoning – in the first stages of an inspection. An inspection begins, of course, with 'the 'phone call'. Inspectors will expect from this very first contact evidence of in-depth and insightful data collection and analysis. Their decision whether to give numeracy 'extra attention' during the inspection in school will depend, in part, on the availability and quality of what school leaders are able to provide in response to the phone call. Guidance for inspectors making that introductory phone call requires that they:

- "*ask for any analysis of national reading and numeracy tests for pupils in Key Stage 2 to be available for scrutiny;*
- *ask for any analysis of additional literacy and numeracy data to be available for scrutiny, e.g. overview of entry with reading ages significantly below chronological and functional literacy, evaluation of intervention programmes, evidence of literacy and numeracy improvements across cohorts and key stages.*

- *If literacy or numeracy require extra attention from inspectors, ask for a high-level analysis of any additional literacy or numeracy data, e.g. an evaluation of literacy or numeracy intervention programmes and any evidence of improvements across cohorts and key stages*" (Estyn, 2013a: extract from list)

Estyn's *Supplementary guidance: Literacy and numeracy in primary schools* includes an Annex describing good practice, 'Annex 4: An overview of good practice in provision for numeracy' (see Appendix 5). This is a valuable resource for schools since it presents an overview with sufficient detail for schools to begin to understand and identify the connected elements of a good numeracy (school-specific) curriculum and high quality teaching for numeracy.

Numeracy in other subjects

Estyn outlines approaches to teaching numeracy as a skill across the curriculum and the opportunities that schools must recognise, exploit and create. Estyn's comments include many references to numeracy. Below are those same comments with Estyn's references to '**numeracy**' replaced by '**numerical reasoning**'. It is helpful to emphasise the importance of numerical reasoning and to consider it explicitly: after all, numeracy cannot be achieved without numerical reasoning.

*In most **primary schools**, the same teacher teaches learners mathematics and other subjects. This teacher is aware of the level of learners' **numerical reasoning** skills and can identify appropriate opportunities to use these skills in other subjects. Primary schools with good standards of **numerical reasoning** make sure that learners apply their **numerical reasoning** skills well in a range of situations. Many of these schools identify the **numerical reasoning** skills learners require in other subjects and help learners to cope with these demands. These schools identify opportunities for learners to reinforce and practise their **numerical reasoning** skills in these subjects. They also organise a range of whole-school projects and activity days that focus on **numerical reasoning**. These activities enable learners to improve their **numerical reasoning** skills further and also highlight the use of **numerical reasoning** in real-life contexts.*

Secondary schools

*Too few secondary schools have well-established strategies to develop learners' **numerical reasoning** skills across the curriculum. Many schools do not make enough use of subjects across the curriculum as practical and relevant contexts for learners to develop and apply their **numerical reasoning** skills. As a result, even when learners have good **numerical reasoning** skills, they often lack confidence in using these skills in unfamiliar contexts and other subjects. In many schools,*

teachers tell learners at the start of the lesson the skills that learners will use. However, many teachers in subjects across the curriculum do not take enough advantage of opportunities to practise and reinforce the skills learners have learned in mathematics lesson when appropriate. In many lessons across the wider curriculum, the ***numerical reasoning*** *demands of a subject may legitimately be pitched below the level of* ***numerical reasoning*** *that learners have already mastered. However, in a minority of lessons teachers of subjects across the curriculum have too low expectations of learners'* ***numerical reasoning*** *skills and set tasks which are too easy, even where more demanding work would be appropriate in that subject. In these lessons, teachers often carry out calculations for learners, even when learners are capable of doing these calculations themselves, or they allow learners to use calculators for basic calculations that the learners should do mentally. Many secondary schools now have a* ***numerical reasoning*** *co-ordinator to develop a whole-school approach to* ***numerical reasoning****. The co-ordinators work with departments to identify the* ***numerical reasoning*** *demands of the subject. Many also provide advice and training on the methods of calculation that are familiar to learners and make sure there is a consistent use of mathematical terminology. This support is most successful when the co-ordinator has sufficient expertise and experience to be able to lead other staff and is given time and resources to carry out the role. However, the extent to which these actions have improved standards of* ***numerical reasoning*** *varies too much. In many schools the initiative has not developed beyond initial meetings or a staff training day and has not had enough impact on the standards of learners'* ***numerical reasoning*** *skills.*

In a few schools, teachers have embedded the initiative across the whole school and have seen an improvement in learners' ***numerical reasoning*** *skills as a result. These schools have gained the co-operation of all departments and plan carefully to make sure that all teachers:*

- *understand the level of* ***numerical reasoning*** *skills they can expect from learners;*
- *look for opportunities in their subjects for learners to reinforce and practise the skills they have learned in mathematics; and*
- *are aware of the methods and strategies used by the mathematics department.*

These successful schools have adapted their schemes of work to make sure learners are prepared for the ***numerical reasoning*** *demands of different subject areas. In many of the schools, the co-ordinator arranges a different* ***numerical reasoning*** *focus each term and produces classroom display materials and documentation to*

support this focus. They also organise whole-school projects and activity days on ***numerical reasoning****. These activities successfully make sure that teachers are reminded of the need to continue to improve learners'* ***numerical reasoning*** *skills and to apply these in a range of contexts. The majority of these schools successfully enter learners for key skills qualifications in Key Stage 3 and Key Stage 4*. (Based on Estyn, 2010:17)

There are many high quality resources for numeracy across the curriculum on the Wales Learning Platform (Hwb) – for example:

'The mystery of the missing dice' (https://hwb.wales.gov.uk/cms/hwbcontent/_layouts/NGFLSolution/MaterialDescription.aspx?LearningMaterialId=25723&lang=en) is a mathematics mystery for upper Key Stage 2 that involves a series of activities with problems to be solved along the way, including calculating volume, measuring time, cracking a code and plotting a route. There are similar projects and activities on Hwb.

In addition, www.nrich.org.uk provides a vast selection of lesson ideas and background information for teachers who are keen to develop learners' problem-solving and reasoning skills.

The National Foundation for Educational Research (NFER) conducted a systematic review of available teaching resources, considering their relevance and value to the teaching of numerical reasoning. Williams, Fowler and Jones, NFER, 2013) identified a number of teaching resources and evaluated different characteristics of each one in relation to the development of numerical reasoning for teachers and learners.

The skills that learners will develop to support numerical reasoning, are skills that will be valuable in many areas of their learning. The NCETM publishes regular online secondary and primary magazines (https://www.ncetm.org.uk/resources/38450) that include suggestions for developing and using mathematics in other subjects, particularly art and history. (Although NCETM resources use the English National Curriculum as a reference point, they are also extremely useful to support the Welsh National Curriculum and LNF.)

Starting points for improvements should be discovered through some process of self-evaluation in school. If your auditing and self-evaluation activities reveal that there are gaps in your school's provision, you will need to develop a plan for the improvement of mathematics and numeracy, including an explicit focus on developing numerical reasoning. Guidance relating to school improvement and curriculum improvement is available and plentiful, for example, from:

- Estyn
- the Welsh Government
- NCETM
- the National College for Teaching and Leadership (NCTL, formerly NCSL) in England
- professional publications (e.g. from senior leaders' associations).

The National Curriculum

It is important to keep in mind the connections with mathematics teaching – much of the knowledge and understanding developed through the National Curriculum in Wales (Welsh Assembly Government, 2008) and the Foundation Phase Framework (Welsh Assembly Government, 2008) will (must) overlap with requirements for numerical reasoning. The Progression Chart takes account of the statutory mathematics curricula for the Foundation Phase, Key Stage 2 and Key Stage 3.

Driving improvement

School leadership: the challenge of implementing change

School and system leaders will be familiar with (non-subject-specific) resources to support them to implement and sustain change, and those are not reproduced here. CPD events for subject leaders, focusing on developing numerical reasoning, will include suggestions for resources for middle managers relating to change and innovation.

It will be helpful for school and system leaders to think about how they can ensure effective implementation through school planning systems to 'hard-wire' teachers' preparedness to address numerical reasoning and how to develop it so that learners' numeracy improves.

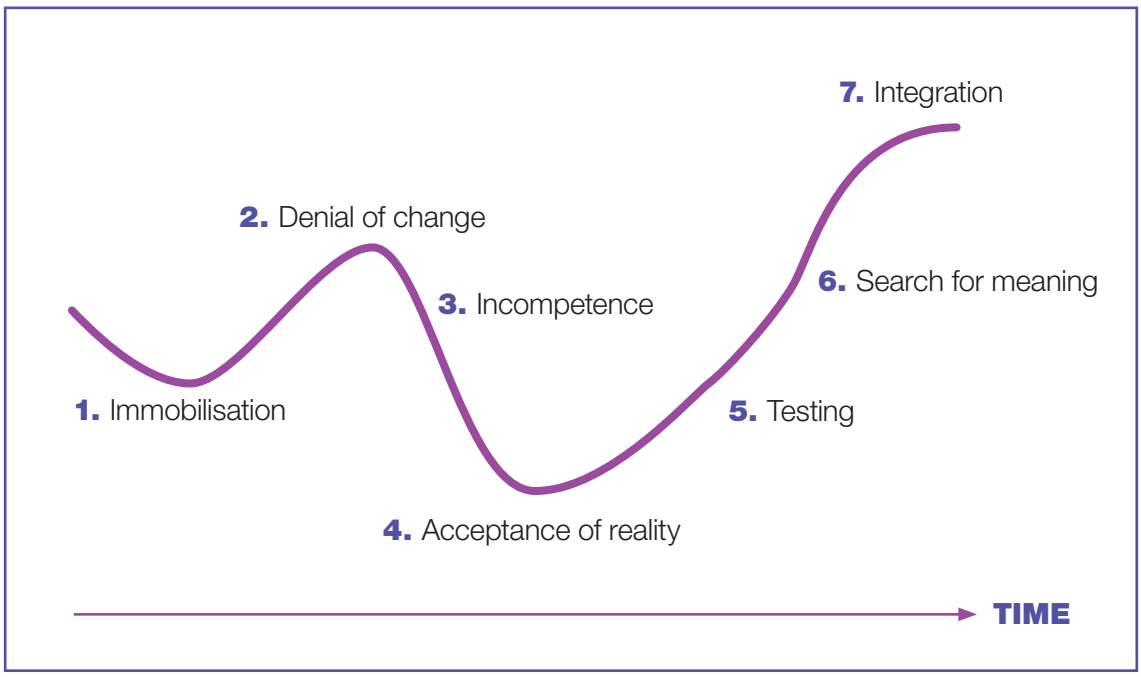
Everett Rogers (2003) (cited University of Warwick, 2014:6) described how different people respond to new ideas and technologies. He defined an innovation as "*an idea, practice, or object perceived as new by an individual or other unit of adoption*"

and diffusion as the process by which an innovation is "*communicated through certain channels over time, among the members of a social system*". As you read the descriptions which follow you might like to consider, for example, how you yourself adopted appliances such as personal computers or mobile phones. Rogers suggests that there are five types of people:

- *"Innovators – risk-takers who are the first to adopt new ideas*
- *Early adopters – willing to try out new ideas but in a more considered or careful way*
- *Early majority – thoughtful people who accept change more quickly than the average*
- *Late majority – sceptics who only change when everyone else has*
- *Laggards – traditional people who much prefer the 'old way'*."

In a similar way people will respond at different rates to organisational change. Individuals' willingness to embrace change will also be affected by their past experiences of change, the extent to which they have voluntarily chosen the change, and their degree of involvement in introducing the change.

Figure 3 Lewis and Parker's transition curve (University of Warwick, 2014)



Lewis and Parker identified seven stages:

1 Immobilisation or shock – a sense of being overwhelmed

2 Denial of change – minimising or trivialising the change

3 Incompetence and depression – with flat performance, frustration, difficulty in coping

4 Accepting reality – letting go of the past and accepting the situation

5 Testing – trying new approaches and behaviours

6 Search for meaning, internalisation – a reflective period with an attempt to understand all that has happened

7 Integration – incorporating new meanings into new and enhanced behaviours.

Complete the table below for yourself and each of your staff:

	YES/NO
Readily takes on new challenges, and tackles them with great energy and enthusiasm	
Responds positively and flexibly when asked to change	
Proactively introduces changes that significantly improve the performance and reputation of their unit	
Implements changes in a planned and coordinated way	
Acts speedily and decisively when planning and implementing change	
Treats people as adults and communicates clearly and honestly with them when introducing changes	
Listens to the genuine concerns of other people and takes account of their concerns when managing change	
Shows courage and tenacity to overcome obstacles and criticism when introducing change	

How will you use the findings to help you lead and influence your staff to commit to Developing numerical reasoning?

Estyn points out that: "*The most successful numeracy initiatives are effective because of the strong leadership of senior managers in the school. These managers drive the initiative well and provide funding for resources and sufficient opportunities for whole-school training. In these cases, all teachers understand that numeracy is an important priority for the school. These schools include programmes to improve standards of numeracy in the school development plan and, where appropriate, in all subject development plans. These initiatives are regularly on the agenda for meetings of staff.*" (Estyn, 2010:20)

The National College for School Leadership (now the National College for Teaching and Leadership) listed some of the qualities that all successful change leaders have:

"A focus on the big picture:

- *Building enthusiasm – for what is great about working in education and for the potential of what could be achieved at the local level;*
- *Being prepared to challenge the status quo – with staff and other stakeholders;*
- *Dealing with different types of change, both the short-term/quick wins and the long-term kind, to do with changing behaviours, attitudes, cultures;*
- *The ability to employ different ways of supporting and working with people, such as mentoring, coaching and modelling;*
- *Empowering others to step up and make a difference;*
- *Knitting together a range of initiatives into one coherent whole;*
- *Managing people with different ideas and objectives and resolving tensions that arise from them;*
- *Understanding people's different responses to change;*
- *Resilience and persistence.*"

(National College for Teaching and Leadership, 2014)

The above quotations, from Estyn and NCTL, refer to working with staff to share your vision and influence them to 'step up' and commit to change relating to new organisational structures; this is certainly one aspect of leading change that school leaders will be familiar with managing. Even more difficult, however, is securing whole-staff commitment to cultural change – understandably hard to achieve since the staff are, of course, themselves part of that culture.

Cultural change – culture for change and/or change of culture

Maughan, Teeman & Wilson carried out a study on behalf of NFER, of the different forms of support that are most likely to encourage teachers to change their practice. They describe 'constructive problem talk', explaining that: "*This dimension of leadership allows leaders to challenge and change entrenched aspects of teacher culture. It is an aspect of leadership requiring problem solving. Leaders name and describe problems in a way that avoids defensiveness, instead encouraging ownership and commitment. The aim is to examine, respectfully, the extent to which beliefs and practices of leaders and teachers inadvertently contribute to a school's problems*." (Maughan et al., 2012:10)

Mourshed et al. make a key observation in their study on improved school systems. They note that the sustaining of new practices is not only about "*changing the explicit structure and approach of a system, but about how teachers think about teaching*". (cited in Maughan et al., 2012:27)

The way teachers think about teaching is, of course, a key influence on the way they teach; also, the beliefs and values that a community of teachers holds actually becomes the culture of teaching in that community. In a school where teachers do not, themselves, perceive value and enjoyment in relation to mathematics, learners are themselves unlikely to perceive it.

There is a further cultural factor that must be addressed if learners are to achieve more in future. Our beliefs and values about 'ability' should be confronted. It is well known to us that children's beliefs about their own potential, and their expectations and aspirations, will all too often become self-fulfilling prophecies. There has been a great deal of research into effect of low expectations on learning and achievement; some of that research has focused on the notion of ability. Dweck (2006), for example, emphasised that ability is not fixed – that it can and does change and keep changing. Teachers should address their own beliefs about ability because, if they have what Dweck labels a 'fixed mindset', their learners are likely to see themselves as having fixed ability; children are far more likely to believe that they can achieve more and understand better if they have teachers who themselves have a 'growth

mindset'. Dweck tells the story of a group of high-school boys in the US who were reduced to tears when they heard that their potential for learning and achievement was very much under their control and they should not feel consigned to a life believing that they are not capable of more.

There is a pervasive culture on a national scale which schools and teachers are part of too, something that is difficult but not impossible to oppose. 'Excusism' is a term coined by National Numeracy to describe an 'I can't do maths' society. Culturally, in the UK, adults are usually not embarrassed or ashamed to declare 'I can't do maths' or 'I hate maths' – it is apparently culturally acceptable not to be able to 'do maths'. (However, people are not usually so happy to admit that they cannot read or write.) Such a culturally acceptable deficit, not surprisingly, becomes a self-fulfilling prophecy and the cycle continues. Even in schools, teachers often convey messages about mathematics and numeracy, sometimes (but not always) unconsciously. Mathematics is perceived as hard, abstract, something that clever people can do; even something that is used as a punishment. Its relevance to other aspects of our lives and our jobs is not made clear by teachers and, too often, they let children develop negative beliefs and values about mathematics.

Where such negative attitudes are allowed to thrive, children are unlikely to develop mathematics knowledge. Yet, without it, they cannot develop numerical reasoning and become numerate. The consequences are therefore not trivial – children who leave primary school with poor numeracy are unlikely to make up lost ground during secondary school and are likely to leave school without a secure grasp of mathematics and how to use it in everyday life. Personally, socially, professionally and financially, their potential is limited. This is, for many, avoidable, and it is the responsibility of primary education to do all it can to avoid it.

Leading curriculum innovation and changing the culture

The leadership of innovation and change occurs at different levels:

- subject/department leadership
- school leadership
- system leadership.

Subject/department leadership is the focus of a separate CPD event in which delegates will consider how to:

- support and develop practice throughout the school
- raise the profile of numeracy in school
- improve attitudes towards mathematics and numeracy
- monitor and evaluate learning and teaching of numerical reasoning.

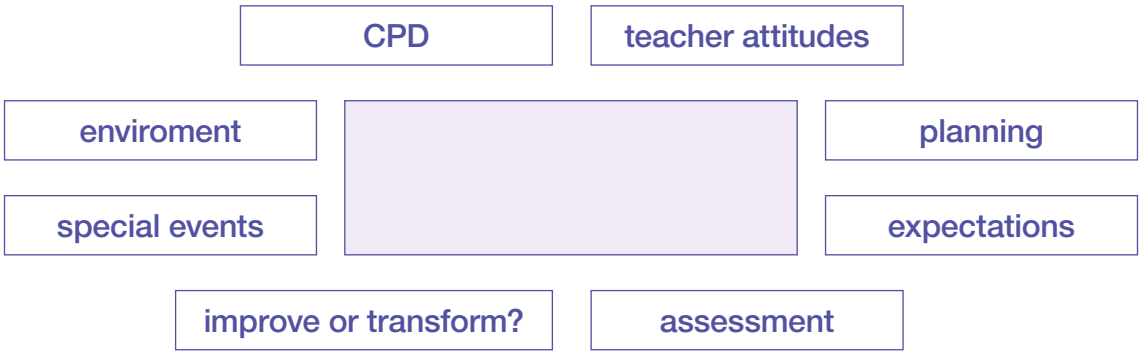
Improve or transform?

"*Transformation is not about improving output or efficiency; it is not about incremental improvement or optimising organizational effectiveness. Transformation is rather about the profound change of every component of the organization following a fundamental reconceptualisation of its purpose and nature … helping leaders review and reconceptualise their personal mindscapes*." (West-Burnham, 2009)

School leaders will need to support and facilitate the work of subject leaders and heads of department, as well as driving and initiating organisational and cultural change at the whole-school, whole-curriculum level.

There must be, at first, a deliberate and focused drive to raise the profile of mathematics and numeracy that is communicated in many ways to all members of the school community. 'Sharing the vision' is not a new idea for school leaders, but that vision should now include explicit references to the development of numeracy (including numerical reasoning). By making clear that initiatives have the full support of senior leadership, changes will be more likely to be implemented and become understood as just another aspect of good primary practice. Such embedding in practice takes time to achieve and development plans must allow for continuing development over a number of years.

Here are some terms that must be considered, in order to develop an effective development plan:



Activity

If 'Developing numerical reasoning' is now written in the shaded box above, how would you complete a concept map to show how all these aspects of school leadership and school life are related to it and to each other? What other boxes would you add?

Driving improvement

School leaders will be familiar with a variety of cycles, processes and checklists for many aspects of school leadership. These will be, to a greater or lesser extent, appropriate for the leadership of curriculum innovation and cultural change.

Maughan et al. reviewed the factors that lead to positive change in primary schools. They found that "*positive change requires strong leadership. Three different leadership practices emerged: strategic, operational and distributive. Taking the first, it was often the vision and high expectations of an effective leader that inspired teachers to change aspects of their working practices. It was also important for leaders to create the right climate for change, as included in our operational label. A third practice is distributed leadership referring to the sharing of leadership responsibilities across an organisation.*" (Maughan et al., 2012:2-3)

Strategic leadership: the vision of school leaders was found to inspire teachers to change their practice [in a number of reports]. For this to work well the leadership should:

- set out a clear and realistic vision – it is important for teachers to understand what the aim of an innovation is if they are to contribute to achieving that aim;
- be based on a well-evidenced rationale, and also adapted to the local context;
- allow teachers to take ownership of the issue, whilst also providing guidance and support where it is needed.

Operational leadership: in addition to a convincing vision, it is also necessary to be able to realise this change practically, by defining the activities that are required. This involves:

- creating the right culture for change, in which innovation is encouraged and sustained;
- developing a climate for learning, in which leaders manage and organise appropriate, in-depth learning opportunities for teachers, over a number of different occasions.

Distributive leadership: it is important that headteachers have an overview of the potential change, but are not the sole drivers. They should ensure that they enable others to also be agents of change.

(Source: Maughan, Teeman & Wilson, 2012)

School and system leaders will find it helpful to self-evaluate their own style and practice against frameworks such as the one outlined above. Maughan et al. also distinguished three categories of practice development: leader-led, peer-led and self-led. Commitment to practice development by senior leadership is important if teachers are to be equipped to enable learners to develop numerical reasoning. Maughan et al. point out that such commitment to the development of teachers' practice can be viewed as a characteristic of an innovative school. They also remind us that "*informal and formal learning opportunities should be backed up with time and funding*." The three types of practice development are:

- "*Leader-led: this can be seen to harness change in schools, impacting on both improved teaching and also on learner outcomes. It can provide the organisational conditions that enable teachers to change. Leadership support is also required for ensuring that other resources, such as space, time and ownership, are available for teachers.*

- *Peer-led: collaboration between teachers working in different schools, or between teachers and researchers, can support effective practice development by making classroom practice visible and providing a form of peer accountability.*
- *Self-led: in this form of practice development, teachers work on their own or with others to reflect on their own practice to stimulate improvement".*

(Maughan et al., 2012:4)

All of these should be evident in schools likely to successfully develop learning and teaching.

Chew and Andrews also describe essential characteristics for practice development:

- mutualism;
- a sense of shared purpose; and
- allowances for individual expression.

(cited in Maughan et al., 2012:17)

The inclusion of these characteristics, they argue, enables teachers to develop as pedagogical leaders.

As well as generic leadership and management resources, there are mathematics-specific resources available, providing focused guidance, developed by subject experts.

For example, National Mathematics Partnership (2011) set out five key levers to raising achievement in primary mathematics:

"Establishing a culture of high achievement for all

- *A belief that high achievement in mathematics is possible and should be the aim for all 11 year olds;*
- *SLT provide the strategic aspiration; the mathematics leader embodies it;*
- *An unerring focus on the progress of pupils – collectively and individually – expecting that all pupils should achieve at least the expectation for their year group from the moment they enter YR to when they leave Y6;*
- *The school will not give up on any pupil, no matter how challenging and time consuming they are.*

Providing a challenging and engaging curriculum for all

- *The curriculum is well-structured with clear and ambitious aspirations for all learners;*
- *The curriculum builds in problem-solving and using/applying mathematics as well as all required concepts, skills and understanding;*
- *The medium-term plans support teachers in maintaining high expectations;*
- *There are minimum expectations for each year group that enable achievement of at least the expectation for their age group for all learners, and higher than this for many.*

Rigorous assessment and tracking of the progress of pupils

- *There is regular monitoring of the progress of all and timely response/intervention established for those falling off the expected trajectory;*
- *Identified strategies and programmes are in place for intervention and support, aimed at getting back on track those falling behind;*
- *All pupils are set ambitious targets and are supported in striving towards them.*

High quality teaching and learning classroom experience

- *Learning is the priority rather than teaching, with a sharp focus on the pupils, their engagement and the progress they make*
- *Good practice is sought, identified and shared – developed with external input where necessary.*

High quality regular teacher development at the whole school and at individual level

- *Regular staff meetings focus on teaching and learning issues in mathematics and sharing developing practice*
- *The school is a 'learning organisation' and engages in effective CPD for mathematics including action research, sharing practice and coaching or mentoring*

- *Debate and development about the pedagogy of the classroom is facilitated through whole-school CPD and at individual level*."

(National Mathematics Partnership, 2014)

Crossley (2013) believes in an 'inside-out approach' – that this is about schools seizing the agenda and working with staff to create optimism, confidence and a sustainable approach to school change and transformation and to inspire and encourage teachers, aspiring leaders and school leaders to have the confidence to make a difference. He believes that, though change is difficult, if the culture is correct, anything is possible. Crossley emphasises the role that all staff must play and that in all schools, including weak and failing schools there are pockets of greatness. He discusses what he calls "*abandonment and redeployment of existing resources*" (p 18). He includes staff of course and a key focus of the book argues that: "*real transformation is not necessarily about new resources but making the most of the resources we have*." (p 2). Crossley summarises: "*… in the final analysis it is about creating a positive culture that is receptive to change and in which people – staff, students and others – feel supported… It is all about culture, engagement and buy-in. However, these are not enough on their own, as what is important is whether a student, teacher, school or system is moving forward and making progress.*"(p 283) (Crossley, 2013)

Action plan – what will you do in school:

With staff, with systems, with resources, with the environment…

- by the end of next week?
- by the end of summer term?
- by Christmas?
- by July 2015?
- ongoing monitoring and evaluation?

(See Appendix 6.)

Auditing current practice and provision

Estyn provides guidance on auditing current provision:

"Any self-evaluation programme should include lesson observations, in addition to those required for performance management purposes, because these observations

provide important evidence about pupils' achievement and progress in learning, and about the quality of teaching and assessment.

A lesson observation programme could include:

- *observation of all staff each year by senior managers;*
- *observation of staff by senior managers as part of an evaluation of a topic or theme;*
- *observations by external visitors such as consultants, local authority advisers or teachers from other schools;*
- *observations by appropriate peers; and*
- *peer observations across departments as part of a theme or to disseminate good practice."*

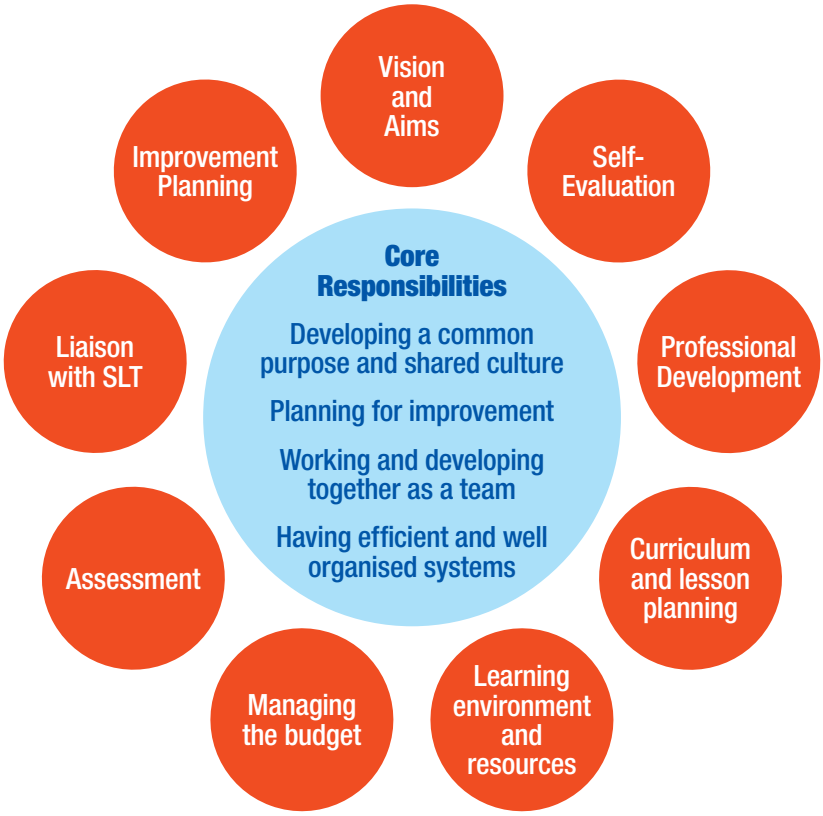
(Estyn, 2011a:5)

School leaders should carry out at least two joint lesson observations, with subject/department leaders, before those leaders attend their 'Developing numerical reasoning' CPD event. This will, of course, form part of your evaluation process as you begin to focus on mathematics teaching intended to develop numerical reasoning. Joint lesson observations should also form part of your plan for ongoing monitoring and evaluation of all strands of the numeracy curriculum.

The NCETM provides resources for self-evaluation at different management levels. Excellence in Mathematics Leadership (https://www.ncetm.org.uk/resources/21289) is intended for middle leaders in primary and secondary schools and provides guidance and support for subject leaders to fulfil all aspects of their very important role. It provides a clear layout of the 'terrain' of effective subject leadership in the form of a 'map' of the key elements of subject leadership and opportunities to explore the four core subject leader responsibilities:

- developing a common purpose and a shared culture;
- planning for improvement;
- working and developing together as a team;
- having efficient and well organised systems.

Figure 4 Key elements of subject leadership (NCETM, 2014)

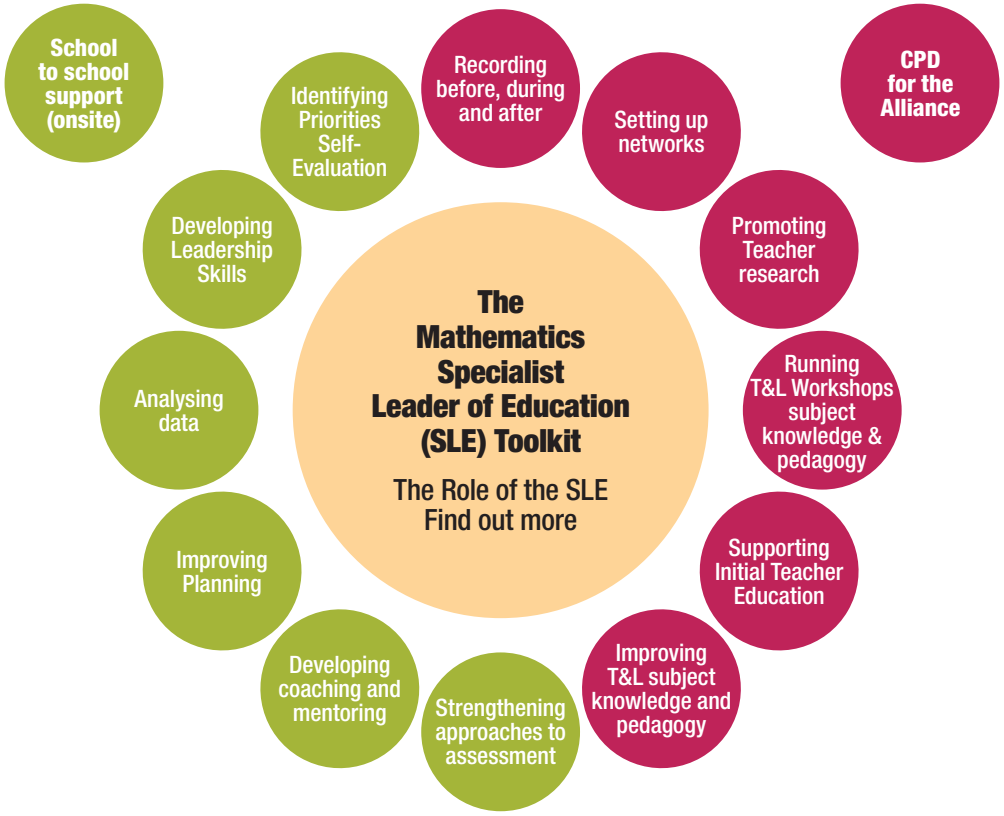


There are descriptors and checklists to help subject leaders improve mathematics learning and teaching (including numerical reasoning) from Level 4 (low) through Levels 3 and 2 to Level 1 (high). (Note: these are labels assigned by NCETM and do not relate to Ofsted or National Curriculum levels.)

Figure 5 contains the 'Core responsibilities' for 'Planning for improvement' at each level. NCETM materials are not written for a Welsh audience but this can be disregarded when the potential contribution of the resource is considered.

At the system level, leaders will also find another NCETM resource supportive. It is the 'Mathematics SLE Toolkit'. (https://www.ncetm.org.uk/resources/38389). Written for the Specialist Leader of Education role in England, this interactive toolkit provides resources for system leaders to facilitate school-to-school support as well as across networks, clusters and chains of schools.

Figure 5 NCETM Mathematics SLE Toolkit



Tests

In May 2014, all learners in Wales from Year 2 to Year 9 will be tested on numeracy through two written tests – one assessing procedural knowledge and one assessing numerical reasoning.

Numerical reasoning support materials, now available on www.learningwales.gov.uk provide teachers with guidance and activities designed to develop learners' reasoning skills for numeracy. The mathematics subject leader training will include sessions focusing on these support materials in more detail, exploring the key reasoning skills referenced.

One example, for Year 5, is 'Blocks':

Reasoning skills required

Identify	Communicate	Review
Learners choose their own methods when solving problems.	They write their own questions.	They review work from other groups and give feedback.

These key skills are matched to the Progression Charts, where they are exemplified with age-appropriate statements to support teaching. Subject leaders will be encouraged to think about effective mathematical learning as the forming and developing of these concepts, as a way of strengthening children's numerical reasoning. Suggestions will be given as to how these key messages can be disseminated to other colleagues back in school.

Roles and responsibilities

Below are extracts from *Five Key Levers to Raise Achievement in Primary Mathematics* from NMP (2011). Decide which are the responsibilities of the headteacher and SLT, and which are the responsibilities of the subject or department leader. Add in other responsibilities that are relevant.

What are the responsibilities of all class teachers? of teaching assistants?

- SLT provides the strategic aspiration; the mathematics leader embodies it.
- The medium-term plans support teachers in maintaining high expectations.
- There are minimum expectations for each year group that enable achievement of at least the expectation for their age group for all learners, and higher than this for many.
- There is regular monitoring of the progress of all learners and timely response/intervention established for those falling off the expected trajectory.
- Identified strategies and programmes are in place for intervention and support, aimed at getting back on track those falling behind.

- Good practice is sought, identified and shared – developed with external input where necessary.
- Regular staff meetings focus on learning and teaching issues in mathematics and sharing developing practice.
- The school is a 'learning organisation' and engages in effective CPD for mathematics, including action research, sharing practice and coaching or mentoring.
- Debate and development about the pedagogy of the classroom is facilitated through whole-school CPD and at individual level.

(NMP, *Five Key Levers to Raise Achievement in Primary Mathematics*, 2011)

Estyn points out that "*Effective school leadership also involves building distributed leadership and ensuring that staff with leadership and management roles make an important contribution*." (Estyn, 2013a:23)

Does your school have distributed leadership? What will you do to introduce/develop it?

Monitoring and evaluation

Will you need to develop new processes and systems for monitoring implementation of the LNF with a focus on Numeracy Strand 1?

The development of numerical reasoning is vital for numeracy, so school leaders need to be confident that Strand 1 is being taught well. It is inevitable that many staff will need support since their own mathematics education is likely to be underdeveloped in this area.

How will you evaluate the impact of CPD that you are probably now planning?

How will you integrate the new teacher support materials coming through related to the tests?

Return to action plan – need to revise?

The task for school and system leaders over the coming months is considerable since the changes required will involve shifts in mathematics pedagogy, curriculum and culture which affect the whole curriculum and the whole community. Mathematics teaching and the development of numeracy will require, for many, a re-conceptualisation, not only of mathematics but of learning generally – teachers need understanding of how concepts develop, become strong and have potential to support future learning. Planning for implementation and embedding of numerical reasoning, as part of the numeracy curriculum, can only be effective if it is planned as an iterative process – repeated revisions and amendments will be necessary as the LNF and new assessment arrangements become incorporated into current practice in schools.

Have key milestones been identified in your action plan?

How will you know key actions have had their intended impact?

Have review dates been agreed?

National support

What will subject/department leaders receive in their training?

In the summer term, mathematics leaders in the Foundation Phase, Key Stage 2 and Key Stage 3 will have the opportunity to attend the first of two half-day CPD events. At these events, they will engage with a number of activities and questions, including:

- Review gap task completed in partnership with school leaders, following CPD for school and system leaders (this session): consider role of lesson observations for future monitoring
- Reflect and develop action plan
- Development of fundamental mathematical concepts
- Understanding progression
- Misconceptions in learning and about learning
- Assessment of numerical reasoning
- Data – tracking and school self-evaluation
- Supporting staff and developing practice; planning CPD
- Calculation policy; Numeracy policy; Mathematics policy
- Case studies
- Examples of numerical reasoning across the curriculum
- Raising the profile of numeracy and numerical reasoning – learning environment and resources

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Appendix 1

What is numerical reasoning and how can teachers support children to develop it?

A Background Paper by Amanda Simpson on behalf of National Numeracy, 2014

Executive summary

'Developing numerical reasoning' is one of four strands in the new national framework, in Wales, for developing numeracy across the curriculum. It is therefore of prime importance, in the view of the Welsh Government, for raising standards of numeracy. A Background Paper (National Numeracy, 2014), which accompanies this executive summary, explains in depth the rationale underpinning our future development work. The Background Paper explores and summarises some of the research literature that relates to numerical reasoning and takes the reader through the stages of establishing definitions for 'numerical reasoning' and 'developing numerical reasoning'. The latter is of course a significant strand of the modern curriculum in Wales.

This executive summary:

- summarises the current education reforms that are pertinent to numerical reasoning
- presents the definitions developed following a review of research literature
- explains some of the key points contained in the definitions
- outlines 'next steps'.

Current reforms

'Improving schools' is the Welsh Government's plan for implementation of system-wide education reforms. The plan states: "*… the quality of teachers is the single most important factor in determining how much is learnt in the classroom*" (Welsh Government, 2012b:15). The programmes developed in the plan are intended to "*help to make every teacher a teacher of literacy and numeracy*" (Welsh Government, 2012b:12).

The National Numeracy Programme (NNP) defines numeracy as: "*Identifying and applying numerical reasoning skills in order to solve a problem, and carrying out the*

numerical procedures which enable people to work out and show their solutions" (Welsh Government, 2012a:2).

The NNP explains that: "*Previous research suggests that reasoning abilities play at least as an important role in* [learners'] *mathematical learning as does their knowledge of mathematical procedures*" (Welsh Government, 2012a:11) and states its intent to assess numerical procedures and numerical reasoning separately.

One of the pillars of the NNP is "*a framework to provide clarity about the skills learners need to master each year*" (Welsh Government, 2012a:16); this is the LNF, which became a statutory curriculum requirement from September 2013, to be provided alongside, and intertwined with, the National Curriculum for Mathematics (Welsh Assembly Government, 2008).

Within the LNF, numeracy is described as: "*the application of mathematical understanding in daily activities at school, at home, at work, and in the community. There is more to numeracy than teaching the rules and procedures of mathematics. … Numeracy describes the set of skills needed to tackle real-world problems in a variety of situations by applying numerical reasoning in order to plan how to solve the problem, and then carrying out the mathematical procedures to find the solution*" (Welsh Government, 2012a:20).

There are four strands making up the numeracy component of the LNF, intended to be taught across all subject areas:

- Developing numerical reasoning
- Using number skills
- Using measuring skills
- Using data skills.

Strand 1, Developing numerical reasoning, is presented in the framework document as three 'elements':

- Identify processes and connections
- Represent and communicate
- Review.

We might think of these elements as the thinking and communication skills needed to solve problems – i.e. to be numerate.

Definitions

Whilst the current reforms give high status to numerical reasoning, the materials produced for schools do not define numerical reasoning adequately for effective use by practitioners. After carrying out a review of literature to help clarify the meaning of 'numerical reasoning', we have developed the following definitions:

Numerical reasoning is:

"*the process of using 'number sense' (i.e. knowing when to use a particular operation; when to use mathematical relationships; the ability to monitor one's performance when computing, for example judging the reasonableness of an answer with respect to an applied problem by what one knows about numbers) which facilitates the formation of conclusions, judgements, or inferences from facts or premises in order to tackle real-world mathematical problems in a variety of situations. For numerical reasoning to be possible, it is necessary that 'powers' are developed and brought to the surface; and also that learners become accomplished at 'knowing-to', so that they are able to respond flexibly and fluently to problems*."

Developing numerical reasoning is:

"teaching and learning that guides and supports the development of skills and conceptual mathematics knowledge such that learners become increasingly able to 'know-to' and 'know-how-to' use their interconnected knowledge and their well-developed problem-solving skills in order to tackle and solve real-world problems."

These definitions will be used in developing resources and guidance for use by practitioners in all year groups from Reception to Year 9. Materials and professional development events will also be created and provided for school and system leaders and for mathematics/numeracy subject leaders.

The **key points** in these definitions that need to be explained here are:

- application of knowledge and skills
- 'knowing-to' and 'knowing-how-to'
- children's powers and bringing them to mind.

Mason was interested in the idea of reasoning and thinking and number sense and reflects on the meaning of the word 'sense'. He reminds us that, as human beings, we instinctively make sense of our world, and that our senses are the media through which we make contact with the world. Sense (or meaning) is achieved by

"*using natural powers to collect, classify, assimilate, accommodate, and even reject sensations, whether physical or imagined, remembered or constructed, literal or metaphoric*" (Mason, 2008:59).

Mason also lists what he calls children's 'powers':

- Imagining and expressing
- Focusing and de-focusing
- Specialising and generalising
- Conjecturing and convincing
- Classifying and characterising.

Mason goes on to note that: "*The fact that every child has displayed these powers does not mean that they are automatically thinking mathematically. Rather, in order to be thinking mathematically, these powers need to be exercised and developed, and for the most part brought to the surface so that they can be used intentionally*" (Mason, 2008:67).

When powers are brought to the surface, or 'brought to mind', an individual is demonstrating that he/she knows to draw upon some aspect of their knowledge. This is what Mason and others refer to as 'knowing-to'.

The concept of 'knowing-to' is related to research about transfer of knowledge which is of course the same as 'application' of knowledge and skills, since it requires re-use of things learned in some other context in order to tackle a new problem. What we can learn about 'knowing-to' is therefore highly pertinent to our understanding about developing children's capacity to reason numerically. Recent and contemporary research presents a view that 'knowing-to' is a skill that enables numerical reasoning – even that it is necessary for it.

The 'learnings' from research by experts must now be integrated into the modern Welsh curriculum for mathematics and for numeracy, so that children in Wales are able to become more numerate.

Research shows us that knowing-to is more than a behaviour that can be trained; it is about awareness and what is attended to (or is not). Knowing-to is a vital aspect of numerical reasoning, and an appropriate learning progression for the development of numerical reasoning must incorporate preparation for knowing-to.

Children's re-use of existing knowledge can only be triggered if they recognise the relevance of it in a new situation. This is only possible if some similarity between the two is perceived (consciously or sub-consciously); this is the key to transfer or application. When we are trying to understand how numerical reasoning develops, we can see that numerical reasoning – that is, application or transfer of mathematical knowledge to new problems can, and must, be prepared for. This entails much more than training children how to react to certain stimuli, but equipping them to be able to 'know-to', with knowledge that is likely to resonate with aspects of other situations – that will be 'brought to mind'.

Testing, CPD and attitudes

A high quality curriculum cannot, on its own, bring about sustained change; it must be developed (initially and following implementation) in conjunction with challenging systems of assessment and support for teachers to develop high quality knowledge and pedagogy. All three of these 'branches' of the wider education system in Wales must be coherent, not only within themselves, but across the whole system so that they all drive improvements in a cohesive, coordinated way. The Background Paper refers to the new (separate) test for numerical reasoning and argues that CPD will be necessary if teachers are to be effective in providing the new curriculum for numeracy.

'Attitudes of mind' are mentioned in the Background Paper, recognising that teachers' attitudes will influence the way that they teach and that children's attitudes will influence the way they learn. This is an area that merits a deeper examination than is possible here, or even in the Background Paper; however, it is valuable to acknowledge the importance of understanding and influencing the attitudes of learners and teachers of mathematics and numeracy. There have been several influential publications in recent years that consider the affective aspects of success in learning (e.g. Claxton, 2002; Dweck, 2006) so these, as well as published research findings about cognition in learning should, and will, be consulted in the development of further guidance for schools and teachers.

The above definitions of numerical reasoning, together with assessments in development, will be used to inform the creation of materials and resources that will enhance teachers' practice and learners' achievements in this area.

The document *Numerical Reasoning: A coherent approach to curriculum provision and professional development. Doing what it takes* (NN & NMP, 2013) proposed a model for transforming education in Wales which simultaneously underpins, and is reflected in, our literature review. Outputs from development work which is

now commencing with the National Mathematics Partnership (NMP) will include progression maps, teacher packs and leadership guides, all of which will be grounded in the rationale and definitions presented in the Background Paper and this executive summary.

Appendix 2

Document E: Prompts for teaching numeracy

Standards

Inspectors should focus on how well pupils can work with numbers and data and how well they can use them in their learning. Pupils' progress will be seen in their skills in using number to solve problems, in analysing information and in making informed decisions based on calculations. It will be seen in their ability to tackle problems in unfamiliar contexts and identify which skills and concepts are relevant to the problem.

How well do learners:

- identify and use an efficient strategy for calculations, including mental methods, written methods and use of a calculator?
- explain their thinking to show their understanding of number processes and concepts?
- demonstrate they have a secure knowledge and understanding of number facts (for example place value, equivalence of decimals and fractions, ordering decimals)?
- demonstrate a sound understanding of calculation methods (for example tables, bonds, mental and written methods and efficient use of a calculator) and calculate accurately?
- demonstrate an awareness of shape, scale, size and position?
- evaluate data to make informed decisions? Are pupils able to collect, organise and analyse data effectively?
- apply their skills accurately when working independently and with others?
- evaluate their solutions?
- cope with the mathematical demands made in the subject?
- draw on skills and concepts learned previously?

Teaching

Points to consider – how well does the teaching:

- plan well for numeracy provision?
- highlight the relevance of using numeracy skills to solve problems?
- identify opportunities to support pupils' number skills and mental agility?
- ensure that pupils are using numeracy skills when faced with an appropriate challenge, and ensure there is an increasing level of challenge in tasks?
- demonstrate good use of language to help the development of learners' numeracy skills?
- make frequent links across the curriculum, so that concepts and skills are developed further by being applied in different, relevant contexts?
- make connections between types of numbers – fractions, decimals and percentages?
- use mathematical information to improve pupils' reasoning and problem-solving skills?
- encourage pupils to talk about and explain their work, look for patterns, interpret and draw valid conclusions from their data?
- use probing questions to improve learners' understanding?
- ask pupils to explain their thinking and help them elaborate on their answers and make learning connections?
- encourage paired and group working, and promote 'active' participation?
- make effective use of techniques where learners support each other's work and check for accuracy?
- exploit the use of ICT to support the development of pupils' numerical and problem solving skills?

(Estyn, 2013, *Supplementary guidance: Literacy and numeracy in primary schools*)

Appendix 3

Document F: Questions for the senior manager overseeing the work of the co-ordinator(s) for literacy and numeracy and/or English and mathematics

Select the most appropriate questions according to lines of inquiry:

1.1.4	What is your view on standards of literacy and numeracy in the school?
2.1	What actions have you taken to promote the development of literacy and numeracy throughout the school? How are you planning to develop learners' skills?
2.1	How well is this literacy and numeracy work co-ordinated and managed? What is the impact of the school's literacy and numeracy policy in helping learners develop skills systematically, over time and in a broad range of contexts?
2.1	Are there any barriers preventing learners developing good literacy and numeracy skills?
2.1	How do you ensure the curriculum provides appropriate opportunities for learners to develop their skills?
2.2.2	How do you track and monitor learners' progress in literacy and numeracy?
2.2.2	Is information on pupils' skills developments shared effectively between phases?
3.2.1	How do you review and evaluate the impact of your literacy and numeracy policy?
3.4	What training and support have staff received to improve literacy and numeracy?

(Estyn, 2013, *Supplementary guidance: Literacy and numeracy in primary schools*)

Appendix 4

Document G: Questions for the literacy/numeracy skills co-ordinator

Select the most appropriate questions according to lines of inquiry:

1.1.4	What is your view of standards of literacy and numeracy in the school?
1.1.4	How much difference are you making to learners' progress and development, in particular learners involved in intervention and support programmes?
2.1	How do you plan to raise standards in literacy and numeracy?
2.1	How do you identify and map skills and develop them progressively?
2.1	What is the impact of the school's literacy and numeracy policy in helping learners develop skills systematically, over time and in a broad range of contexts?
2.1	What factors are preventing learners developing good literacy and numeracy skills?
2.1	What is the format and frequency of catch-up sessions?
2.2.2	Do you know how well learners are progressing, including those receiving targeted support or extension?
2.2.2	How do you identify the learners who need support to improve their literacy and numeracy skills?
2.2.2	How is the progress of learners on the intervention programmes communicated to managers and other staff?
3.2.1	How do you ensure that all staff teach the many concepts and methods with consistency?
3.2.2	How effective are your intervention strategies in helping learners catch up with their peers? How do you review and evaluate the impact of literacy and numeracy initiatives?

3.2.2	How do you ensure that classroom teachers are aware of the teaching and learning strategies and the resources used in the intervention programmes?
3.3.1	What are you doing to improve the development of learners' skills during transition?
3.4	What training do you provide for support staff, learning coaches and peer buddies so there is a consistent approach to the development of learners' skills?

(Estyn, 2013, *Supplementary guidance: Literacy and numeracy in primary schools*)

Appendix 5

Excerpt from 'Annex 4: An overview of good practice in provision for numeracy'

Schools that promote numeracy well provide pupils with regular opportunities to use and apply their mathematical skills in a range of situations and contexts. Plans clearly demonstrate the progressive development and reinforcement of pupils' numeracy skills and mathematical language across a range of areas of learning.

Schools that co-ordinate the provision for numeracy effectively:

- identify areas of the curriculum where pupils have opportunities to use and apply what they have learned in mathematics; and
- ensure pupils are taught the mathematical skills and knowledge they need to meet the requirements of other curriculum areas.

In the **Foundation Phase**, there should be a balanced approach to teaching numeracy, including:

- number awareness – counting, recognising, reading, writing and ordering numbers, recognising patterns and relationships;
- calculating – understanding and using a variety of methods to solve number and word problems;
- money – recognising, sorting and using coins, finding totals and giving change;
- measures – comparing, ordering and recognising the need for standard units;
- shape, position and movement – recognising and using 2D and 3D shapes, understanding and using vocabulary to describe position and movement; and
- handling data – sorting, classifying, collecting, organising, recording, interpreting, and extracting data from an increasing range of sources.

Using continuous provision to develop pupils' numeracy skills

- role play – clock, scales, coins and notes, timetables, measuring tape, calculator;
- sand and water – calibrated apparatus for measuring;

- construction – variety of blocks/boxes of different shapes and sizes, rulers and tape measures, plans and photographs of buildings;
- a book area – counting books, number rhymes, books that emphasise mathematical language, such as positional language, sequencing or ordering and comparing; and
- creative area – variety of resources and materials that can be used for pattern making.

What would you expect to see?

- a rich and dynamic indoor and outdoor environment, where numeracy is given high status;
- plenty of good quality opportunities for pupils to apply their numeracy skills in all areas of learning;
- mathematical area, ICT area and play areas that are well equipped with resources that enhance pupils' numeracy skills;
- displays of good quality that illustrate the forms and purposes of number;
- teachers providing frequent opportunities for pupils to 'use and apply' mathematics in their daily work, including improving the level of challenge for the more able to develop their thinking and problem-solving skills;
- pupils undertaking work in mathematics across a range of contexts and situations;
- all staff, including support staff, involved in drawing up mathematical development plans and short-term plans for teaching mathematics and numeracy; and
- teachers discussing with pupils how they can use their skills in mathematics effectively to solve problems and record their findings in different contexts and situations, and developing an ethos of 'numeracy' where pupils see mathematical inquiry as a natural part of learning.

In **Key Stage 2**, pupils should build on the skills, knowledge and understanding acquired in the Foundation Phase. There should be a balanced and progressively challenging approach to teaching numeracy, including:

- continuing to develop pupils' use of the number system, moving to calculating fluently within all four number operations;
- developing pupils' mental calculation strategies alongside their written methods;
- developing pupils' estimation and checking strategies;
- developing pupils' accuracy when measuring;
- continuing to develop pupils' knowledge of 2D and 3D shapes and their properties;
- providing opportunities for pupils to collect, represent, discuss, interpret and explain data from a variety of sources; and
- developing pupils' ability to reason and communicate mathematically, using appropriate mathematical language.

What would you expect to see?

- a rich and dynamic learning environment where numeracy is given high status;
- plenty of good quality opportunities for pupils to apply their numeracy skills in all areas of the curriculum;
- all practitioners, including support staff, are involved in the initial stages of planning for numeracy;
- displays of good quality and visual prompts that illustrate the forms and purposes of mathematics;
- the use of real-life purposeful investigations to solve mathematical problems;
- plans that progressively develop pupils' understanding of 'number' and 'shape and space', in ever-more challenging and complex ways using high quality resources;
- schools planning effectively for the progressive development and reinforcement of pupils' numeracy skills and mathematical language across a variety of practical and relevant contexts including the outdoors;
- staff challenge pupils' thinking and understanding by asking probing questions;

- staff use appropriate resources and images to develop pupils' mental calculation strategies to enable them to quickly recall simple number facts;
- pupils encouraged by all staff to use a range of checking strategies including mental estimation, approximation and inverse operation;
- schools quickly and accurately identify pupils in need of additional support for numeracy and providing specific programmes to address these concerns; and
- staff who are good mathematical role models for problem solving and the use of mathematical language.

(Estyn, 2013, *Supplementary guidance: Literacy and numeracy in primary schools*)

Appendix 6

Action planning for Developing numerical reasoning

What do you plan to do/change regarding … ?						
	staff	systems	resources	environment		
What do you plan to do by … ?						
… the end of next week?						
… the end of the summer term?						
… the end of 2014?						
… July 2015?						
… ongoing (looking to the future)?						

Appendix 7

Collaborative interim task

(for simplicity, SL = subject leader)

Resources	Activity	Follow-up
Five Key Levers to Raise Achievement in Primary Mathematics. http://www.nmpartnership.co.uk/resources_primary.htm + consider other resources from the session	Share work done in session with SL	SL will develop work on *Five Key Levers* document in SL CPD in June and October.
Action planning (Appendix 6)	Share and develop: • Have key milestones been identified? • How will you know key actions have had their intended impact? • Have review dates been agreed?	SL to bring to June CPD session
At least one joint lesson observation focusing on developing numerical reasoning	• Agree lesson observation proforma • Sit together during lesson	Reflect together and (SL) write summary – bullet points (three key strengths and three areas for development) SL to bring summary to June CPD session

National Numeracy

Ariennir gan Lywodraeth Cymru
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